



## Combined Natural Convection of a Drink Can using Three-Dimensional Simulation

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### **Abstract**

A drink can placed in a refrigerator is cooled by natural convection is investigated. In this study the full combined boundary layer system on the can wall is simulated. The cylindrical can filled with water ( $Pr=7.0$ ) at temperature  $T_o=20^\circ\text{C}$  is located within a larger cylindrical container filled with air ( $Pr=0.7$ ) at temperature  $T_a=5^\circ\text{C}$ . The container and can have the height-width ratio of 1 and 2, respectively. The walls of the can are very thin, hence, the assumption of zero thermal resistance at walls is used and the heat capacity in the walls is neglected. The outer container walls are maintained at constant temperature  $T_w=5^\circ\text{C}$ . The development of the flow and cool down of the fluid in the can is simulated by solving the governing Navier-Stokes and temperature transport equations using the commercial solver FLUENT. The study examines the placement of the inner can in two configurations. The first case has the inner can placed vertically and the second case has the inner can placed horizontally. Both of the cans placed in the middle of the outer container. The flow behavior of both cases will be presented. The numerical results show that the second case has a rate of cooling around 4.25% faster than the first case.

**Keywords:** Natural convection cooling, full combined boundary layer system, cooling rate.

### **1. Introduction**

One of the most common shapes involving in natural convection heat transfer processes is the cylindrical container, such as thermal energy storage systems, food and drink cans. The simplest model of natural convection in such a container assumes that the vertical wall is maintained at a uniform temperature, with an adiabatic top and bottom. The container is

normally vertically oriented. In most of the earlier studies, the heating process in the container has been studied [1-6]. Recently, the natural convection cooling process in the cylindrical container has been extensively investigated by Lin & Armfield [7-11]. The transient flow patterns show that the flow activity occurs mainly in the vertical thermal boundary layer along the vertical wall and in the horizontal



region at the bottom of the container. Travelling waves observed in the vertical thermal boundary layer and the cold intrusions in the horizontal region were analyzed. The evolving process and flow characteristics are similar to those observed in cavities with differential side-wall heating, for example in Patterson & Armfield [12] where a number of scalings were developed to describe the start-up and fully developed flow structures.

The case of interest here is a drink can placed in a refrigerator. In this case the can has neither isothermal nor isoflux boundary, as the cool air in the refrigerator interacts with the warmer fluid in the can via the combined system consisting of natural convection boundary layers on both the liquid and air sides of the can wall. To better understand this flow, we previously investigated the combined natural convection cooling of a drink can, simulating the full refrigerator/can system with axisymmetric two-dimensional models [13, 14]. Two configurations were examined: the first had the inner can placed vertically in the middle of the outer container with no contact with the outer container walls, and the second had the inner can placed vertically at the bottom of the outer container. In both cases the basic flow structure within the can is similar; with the development of the natural convection boundary layers adjacent to the can wall obeying the same scaling relations, but with very different scaling constants, in particular the total cool down times. The results also compared to simplest models with an isothermal can boundary condition described above. The combined boundary layer solutions had total cool down times more than an order of magnitude greater than the simplest

model solutions, showing that the isothermal boundary condition does not provide a good approximation of the cooling time for the combined boundary layer solution. The non-axisymmetric three-dimensional models were also carried out and compared [15]. The three dimensional models have similar general flows and a rate of cooling around 3% slower than the corresponding two dimensional models.

In this study, the full combined systems are simulated with non-axisymmetric three-dimensional models. The effect of the can placement, horizontal and vertical orientation, will be investigated.

## 2. Numerical Method

### 2.1 Models

The case under consideration is the natural convection cooling of the water in a cylindrical can located within a larger vertical cylindrical container of air initially at a lower temperature than that of the water in the can. The outer container walls are maintained at constant temperature  $T_w=5^{\circ}\text{C}$ . The walls of the can are assumed to be very thin, hence, the assumption of zero thermal resistance at the can walls is used and the heat capacity in the walls is neglected. The cylindrical container has a height  $Z$  and radius  $R$ . The inner cylindrical can has a height  $H$  and radius  $L$ , as illustrated in Fig. 1. The height width ratios  $A$  of the container and can are 1 and 2, respectively. The configuration and control parameters have been chosen to match a can 13 cm high, filled with water ( $Pr=7.0$ ) at  $T_o=20^{\circ}\text{C}$ , and placed in a refrigerator

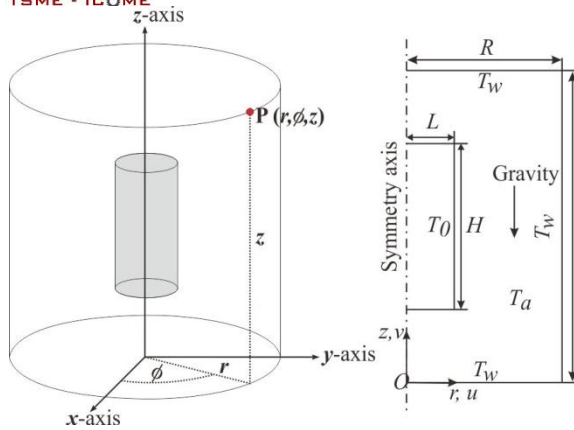


Fig. 1 The coordinate system for natural convection cooling of a can.

containing air ( $Pr=0.7$ ) at  $T_a=5^\circ\text{C}$ . Based on the physical properties of the water and the air, the water domain has  $Ra_w=4.62 \times 10^8$  and the air domain has  $Ra_a=4.20 \times 10^6$  for the can with vertical orientation. The interests of this study are the flow behaviour of both fluids and the effect of the location and orientation of the inner cylinder can. Inner cans are investigated in two separate cases, with vertical orientation (case 1) and horizontal orientation (case 2), as shown in Figs 2(a) and 2(b), respectively.

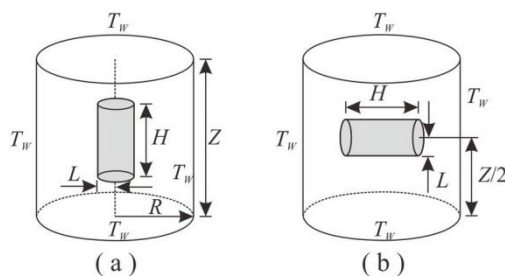


Fig. 2 Flow configurations for natural convection cooling problems of (a) case 1 and (b) case 2.

## 2.2. Governing equations

The unsteady natural convection flow in the cylindrical container and can is governed by the Navier-Stokes equations and the temperature equation. With the Boussinesq

approximation, the equations can be written in non-dimensional form, as follows:

$$\nabla \cdot u = 0, \quad (1)$$

$$\frac{\partial u}{\partial \tau} + u \cdot \nabla u = -\nabla p + \frac{Pr}{Ra^{1/2}} \nabla^2 u + Pr\theta, \quad (2)$$

$$\frac{\partial \theta}{\partial \tau} + u \cdot \nabla \theta = \frac{Pr}{Ra^{1/2}} \nabla^2 \theta. \quad (3)$$

The gradient operator is given by,

$$\nabla \equiv \frac{\partial}{\partial r} + \frac{1}{r} \frac{\partial}{\partial \phi} + \frac{\partial}{\partial z}. \quad (4)$$

The Prandtl number,  $Pr$ , and the Rayleigh number,  $Ra$ , are defined as,

$$Pr = \frac{\nu}{\alpha}, \quad (5)$$

$$Ra = \frac{g\beta\Delta T h^3}{\nu\alpha}, \quad (6)$$

where  $g$  is the acceleration due to the gravity,  $\beta$  is the coefficient of thermal expansion and  $\nu$  is the kinematic viscosity. The length scale are  $h = H$  for  $Ra_w$ , and  $h = Z$  for  $Ra_a$ .

## 2.3. Discretisation

The governing equations are solved with a finite-volume discretization process, carried out using the commercial solver FLUENT 6.3.26. The unsteady segregated solver was used with a second-order upwind scheme for convective terms and a second-order central scheme for the diffusion terms. The PREssure STaggering Option (PRESTO) scheme has been employed for pressure discretisation and the SIMPLE algorithm has been used for the pressure-velocity coupling. A non-staggered mesh, very fine near the walls and relatively coarse away from the walls, is used in this study. The computational domains were generated in GAMBIT 2.3.16 using the Cooper volume meshing scheme for the inner can, resulting in 556,100 hexahedral cells. The Cooper volume meshing scheme was also used to generate the

air domain of case 1 resulting in 1,232,990 hexahedral cells as shown in Fig. 3(a). The air domain of case 2 was generated using the *TGrid* scheme, which created 1,111,912 mixed cells as seen in Fig. 3(b). The Non-Iterative Time Advancement (NITA) was used for both cases. The convective and diffusive terms were discretised with second order accurate schemes. The fractional step method was used to coupling the velocity and pressure. The time step  $\Delta\tau=5\times 10^{-5}$  was used with both cases in this study. The 3D-model of case 1 was validated by comparing the numerical results to those of its 2D-model[13-15]. The results show that the 3D-model produced almost identical to those of the 2D-model, the relative L2 norm <1%.

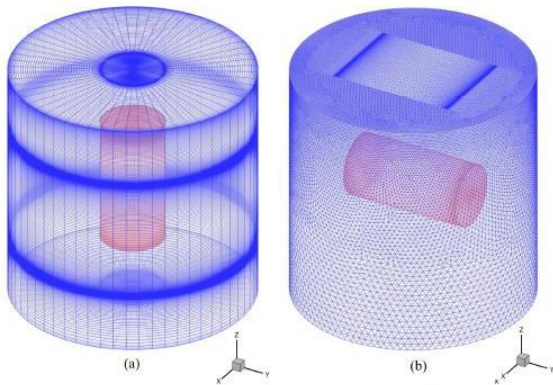


Fig. 3 Three-dimensional models and mesh structures of (a) case 1 and (b) case 2.

### 3. Numerical Results

#### 3.1 Case 1

The walls of the outer container have a constant non-dimensional temperature of  $\theta_w=-1$ . Initially both fluids are at rest and the temperatures of air and water are  $\theta_a=-1$  and  $\theta_o=0$ , respectively. Immediately after initiation of the flow the temperature difference across the can wall leads to the development of natural convection boundary layers on both sides of the

wall. The conduction heat transfer through the can wall from the inner (water) side to the outer (air) side results in cooled water immediately adjacent to the wall on the inner side, with an associated descending natural convection boundary layer. On the outer side, heated air immediately adjacent to the wall, with an associated ascending natural convection boundary layer. This is qualitatively identical to the process of the 2D-simplest model observed by Lin & Armfield [7, 8,10] in their investigation of the natural convection cooling of the fluid in a can subjected to an isothermal wall boundary condition. After the full development of the thermal boundary layer, warm air moves upwards to the ceiling of the container. When it reaches the ceiling, a warm intrusion forms and moves along the top and towards the vertical wall of the container, with a vortex at the front of the intrusion, as seen in Fig 4 at  $\tau=0.5$ .

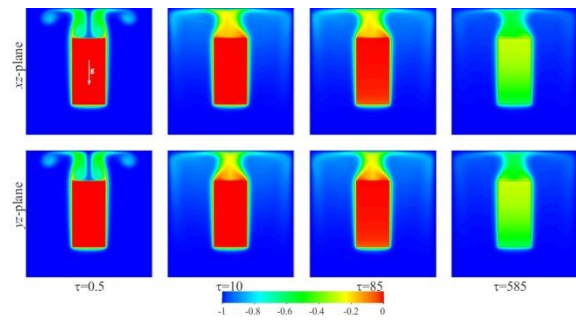


Fig. 4 Time evolution of transient temperature of case 1.

After that, the stratified air starts to fill the container and the air approaches quasi-steady state. At  $\tau = 10$ , the plume of warm air, which discharges from the top of the can towards the ceiling of the container, has moved out of the  $z$ -axis resulting in asymmetrical flow in the air domain.

As the flow in the air domain approaches a quasi-steady state, a thermal

boundary layer gradually forms in the water within the can, the slower rate of formation being a result of the larger Prandtl number. Particle tracking in the air and water domains in Fig. 5 reveals non-axisymmetric spiral motions in the upper regions of both the air and water domains. These non-axisymmetric features persist only during the initial phase of the flow, at later time the fluid in both domains becomes continuously stratified and the non-axisymmetric motions die out.

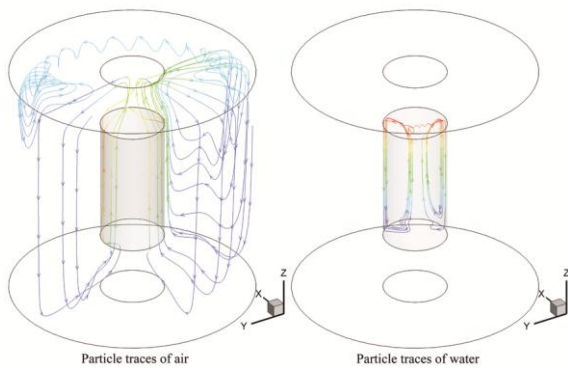


Fig. 5 Particle traces of the flow in case 1 at  $\tau=85$  and the temperature ranging from -1 to 0 represented by the colour of contour lines.

### 3.2 Case 2

Another placement configuration for the can in the fridge is the horizontal one. The main purpose of studying these two problems is to observe the general flow structures and to obtain the cool down times. Fig. 6 shows the evolution of the fluid flow during the cooling process in case 2 on the  $xz$ - and the  $yz$ -planes, respectively. The thermal boundary layer forms rapidly around the can and moves up towards the top edges of the can. After the full development of the boundary layer, the warm air is discharged from the top edges of the can, resulting in a plume of air with counter rotating vortices on the  $xz$ -plane. This start-up phase

has a symmetrical flow on both planes until the

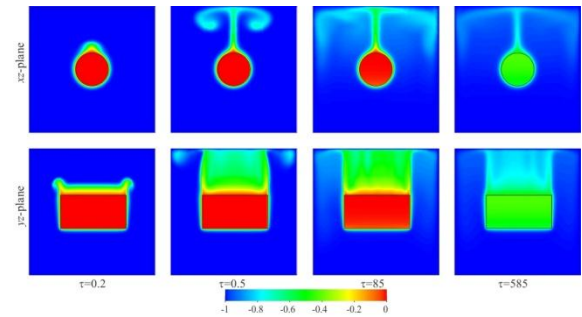


Fig. 6 Time evolution of transient temperature of case 2.

intrusion of the warm air hits the wall of the container. At  $\tau=0.5$ , the flow starts to become asymmetrical as the intrusions on both planes arrive at the container walls at different times. The air starts to fill the container with a chaotic motion with an aperiodic flapping of the plume in both the  $xz$ - and  $yz$ -planes. Similar to case 1, the chaotic and flapping motion decays in the later stages of the cooling process.

The isosurface temperature  $\theta=-0.37$  in the air domain, as seen in Fig. 7 shows the instability of the plume. However, most of the flow near the can wall is stable except at the top edges of the can. In the meantime, the boundary layers inside the can gradually form on the wall and at both ends of the can and they travel down towards the bottom of the can. After that, stratified water continually fills the can.

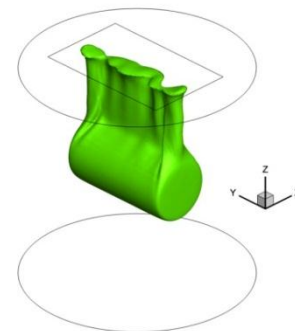


Fig. 7 Isosurface of temperature  $\theta=-0.37$  of case 2 at  $\tau=85$ .

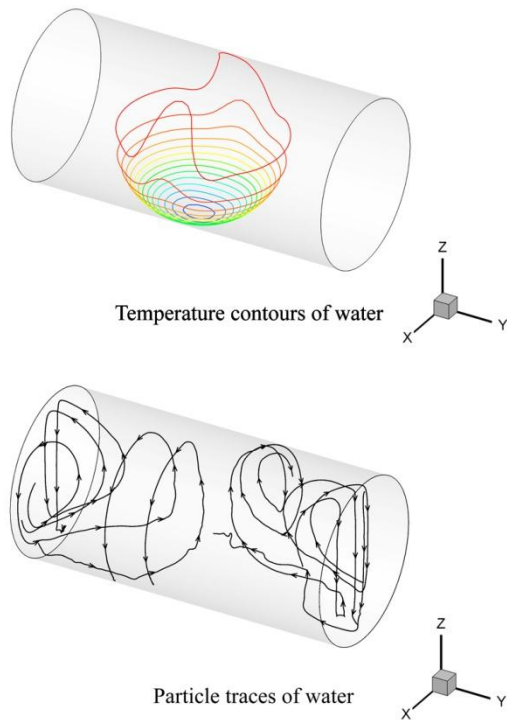


Fig. 8 The temperature contours ranging from -0.77 to -0.99 and particle traces of the water in the can of case 2 at  $\tau = 85$ .

Fig. 8 shows that the structured flow of the water in the can is more stable than that of the air around the can. The stratified water is clearly symmetrical on the  $xz$ -plane. The particle traces in Fig. 8 also confirm that the flow in the water domain of case 1 is quite smooth, even right under the area where warm air is discharged. Unlike the vertical can case, the curvature of the can wall reduces the chaotic motion of the air on top of the can, resulting in a more uniform behaviour in the water underneath.

### 3.3 Cooling rate

The cool down times of water in the can are compared, where the cool down time is the time required for the average temperature in the can to reach the value  $\theta_{w,avg}(\tau_d) = -0.99$ . Using the scaling relation of the simple model with

constant temperature on every wall, as considered in [16], the cool down time of the horizontally placed can was observed to be 19% faster than that of the vertically placed can with the same can aspect ratio of 2, as used in this study. The study involving can placed in the middle of the container, the simulation results of the fully combined boundary layer 3D-models show that the non-dimensional cool down times,  $\tau_d$ , of case 1 and case 2 are 19304.3 and 18479.9, respectively. The horizontally placed can have a rate of cooling around 4.25% or 75 minutes of the dimensional time faster than the vertically placed can. Since the can is placed horizontally, the height of the can is reduced, thereby reducing the time required by stratified water to fill the whole can. The results of this study correspond to those of Varma and Kannan [17, 18] showing that the horizontally placed can with the constant temperature on the wall has a heating rate around 10% higher than the vertically placed can.

## 4. Conclusions

The three-dimensional models of the case with horizontal inner can placement configurations were compared to the cases with vertical inner can placement configurations. Result showed that the flow of the air domain of over the can was more chaotic and more flapping than that observed in the vertically placed can cases. The most unstable flow occurred with the horizontal can case as this created the largest gap between the top of the can and the ceiling of the container. The multi-column air plume above the can was also observed in case 2. The cool down time shows



that the horizontally placed can in the middle of the container cooled at a rate of 4.25% faster than the vertically placed can.

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